Optimal Allocation: Design without Transfers

Elementary school choice in Boston (2012)

- Students rank any number of programs within their zone + walk-zone.

- Schools priorities over students:
  1. Continuing
  2. Siblings
  3. Walk-zone (applies to only 50% of seats)
  4. lottery number

Main entry grade K2:
- 77 schools
- 123 programs
- ≈20-60 seats/program

Assigning students to schools

Student-proposing Deferred Acceptance (Gale-Shapley 62):

While no more students apply

- Each unmatched student applies to the next school on her list.
- Any school that has more proposals than capacity rejects its least preferred applicants

In Boston preferences of schools over students are determined by priorities and lottery numbers

Criticisms of choice plan

HOME / NEWS / EDUCATION / K-12

The Boston Globe

GETTING IN | INSIDE BOSTON'S SCHOOL ASSIGNMENT MAZE

Selection process starts with choices, ends with luck

In towns across America, families buy homes knowing exactly where their kids will go to school. Like their post office, their parish, and their neighborhood pub, it's usually the closest one.

Unpredictable

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The Boston Globe

GETTING IN | INSIDE BOSTON'S SCHOOL ASSIGNMENT MAZE

The high price of school assignment

By Jenna Russell and Stephanie Ebbert
Globe Staff / June 12, 2011

E-mail | Print | Reprints | Text size – +

Like an army of yellow ants, they march across the city: 691 school buses carrying 32,221 students.

They will cost the Boston public schools a staggering $80 million next year, approaching 10 percent of the total school budget.

Unsustainable Transportation Costs

The Boston Globe

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GETTING IN | INSIDE BOSTON'S SCHOOL ASSIGNMENT MAZE

A daily diaspora, a scattered street

Every morning, children in Boston disperse to schools all over. Childhood chums, and neighborhood feeling, can be left behind

By Stephanie Ebbert and Jenna Russell
Globe Staff / June 12, 2011

Scatters Neighborhoods

Fundamental tradeoffs

- **Limit busing**.
- **Efficiency**: Match families with what is best for them.
- **Equity**: Families have reasonable chances regardless of home location or socio-economic status.

- Other considerations:
  - Predictability
  - Simplicity
  - Community cohesion

Outline

- A generalized model, applicable in more settings.
- Characterizations of *"good"* mechanisms in large markets
  - Characterizations for cardinal and ordinal mechanisms
- Apply the theory to Boston school choice (from large to finite)

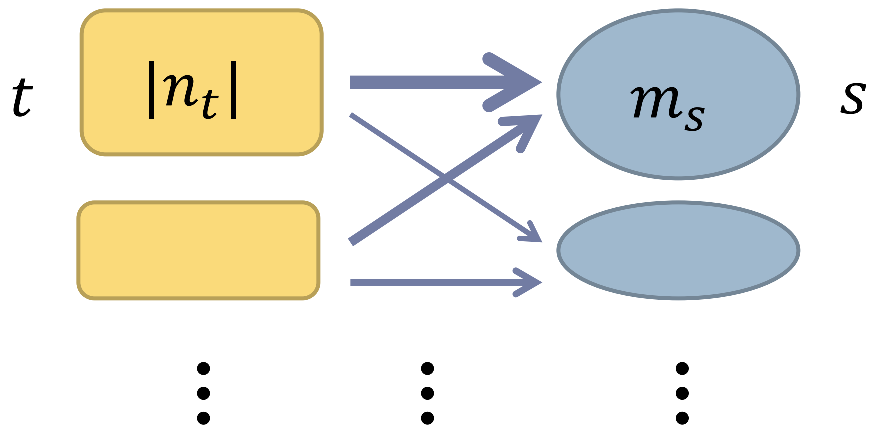
Abstracting key issue from School Choice Example

- Limited resources.
- Private information.
- Balancing efficiency, equity, and system costs.

- Normally use auctions or queues. But money or costly signals cannot be used here.
- Other examples:
  - Course allocation.
  - Lotteries for on-campus housing.
  - Internal allocations of tasks in a company.

Large market model

- Finite set T of agents types.
- Mass n_t of **agents** of type $t \in T$.
- Finite set S of **services**. Service $s \in S$ has capacity m_s.
- Each agent must be assigned 1 service.



$$u_{is} \sim F_{t(i)}$$

Private utility of agent i

Common knowledge prior (regularity conditions: continuous and full relative support)

- Social planner needs to **allocate** services **to optimize a public objective without** the ability to differentiate agents via requiring **monetary payments** or **costly effort**.

Two types of mechanisms

- Cardinal (unrestricted)
- Ordinal:
  - Can only elicit preference rankings, but not intensities. *i.e.* service a > service c > service b.

Related Work

- **School choice mechanisms:** Abdulkadiroğlu, Sönmez 03; Abdulkadiroğlu, Pathak, Roth, Sönmez 06; Abdulkadiroğlu, Pathak, Roth 09; Dur, Kominers, Pathak, Sönmez 13.
- **Assignment mechanisms:** Hylland, Zeckhauser 79, Bogomolnaia and Moulin 01, Budish 11, Mirralles 12, Ashlagi, Shi 13
- **Allocation in large markets:** Thomson, Zhou 93, Zhou 92
- **Lotteries in school choice:** Pathak, Sethuraman 11; Erdil, Ergin 08.
- **Large market analysis of matching markets:** Abdulkadiroğlu, Che, Yasuda 08; Che, Kojima 11; Liu, Pycia 12; Azevedo, Leshno 10; Budish, Cantillon 12, Ashlagi, Kanoria, Leshno 13, Lee 12
- **Implementing assignment probabilities:** Budish, Che, Kojima, Milgrom 12.
- **Diversity:** Konimers, Sönmez 12; Erdil, Kumano 12; Echenique, Yenmez 13.
- **Burning mechanisms:** Hartline, Roughgarden 08, Chakravarty 10

Cardinal interim allocation rule

- An interim allocation rule for type t: maps reported utilities of an agent of type t to assignment probabilities

$$x_t : \mathrm{U} \rightarrow \Delta$$

Utility space

Feasible assignment probabilities

- Incentive Compatibility (IC):
  - $x_t(\boldsymbol{u})$ maximizes expected utility $v(u') = \boldsymbol{u} \cdot x_t(\boldsymbol{u}')$
- Pareto efficient within type:
  - Does not exist $x_t{}'$ with same average allocation as x_t but strictly Pareto improves x_t.
- "Valid": both IC and Pareto efficient within type.

Problem formulation

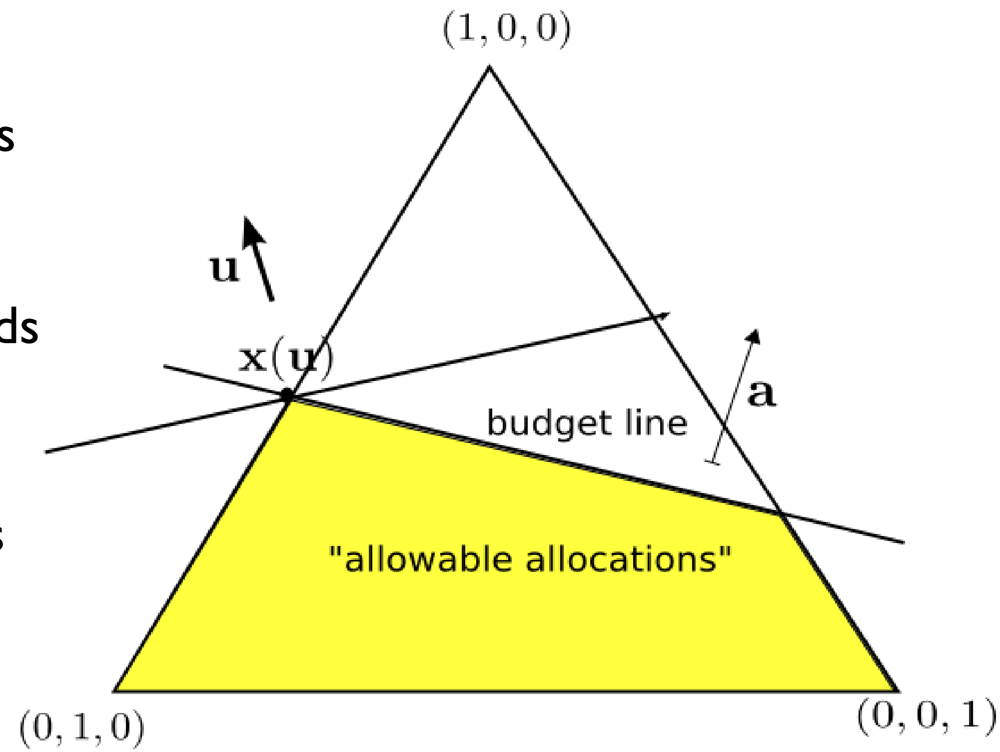
- Given utility priors F_t for each type.
- Max $W(x)$
  - Can encompass social planner's balancing of welfare, equity, system costs, and distributional preferences.
- Subject to
  - x_t valid (incentive compatible and Pareto efficient within type)
  - System costs or distributional constraints.

Characterization of valid allocation rules

Theorem: Let F be continuous and with full relative support for all types. Every valid allocation rule x can be supported as a Competitive Equilibrium from Equal Incomes (CEEI) with some price vector $a \in (0, \infty]^{|S|}$.

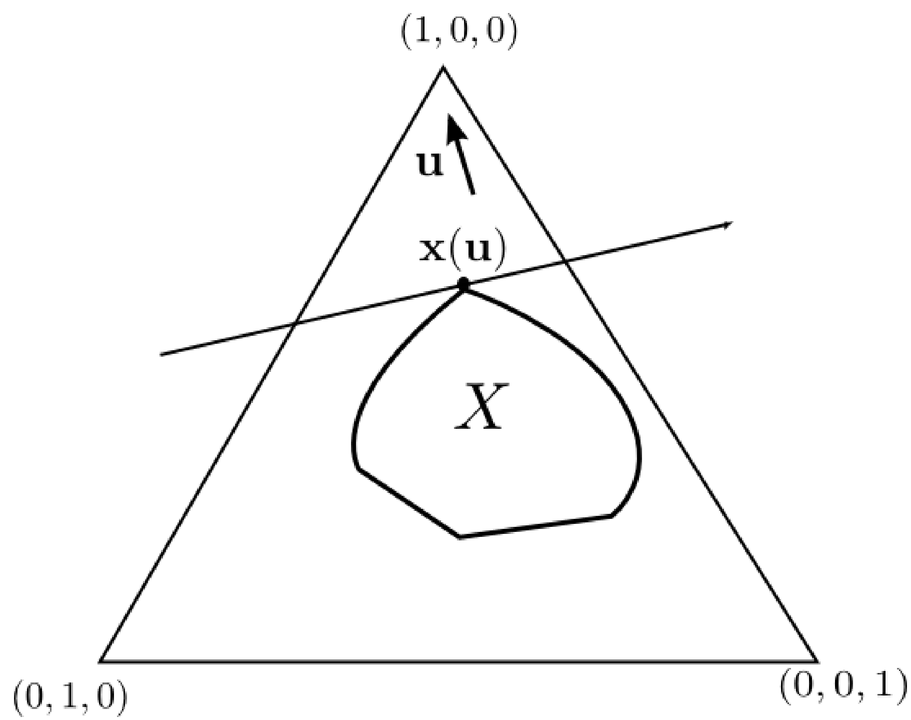
Interpretation: each agent has one unit of budget and can purchase any probabilistic assignment that does not exceeds the budget.

Optimization – only variables are virtual prices



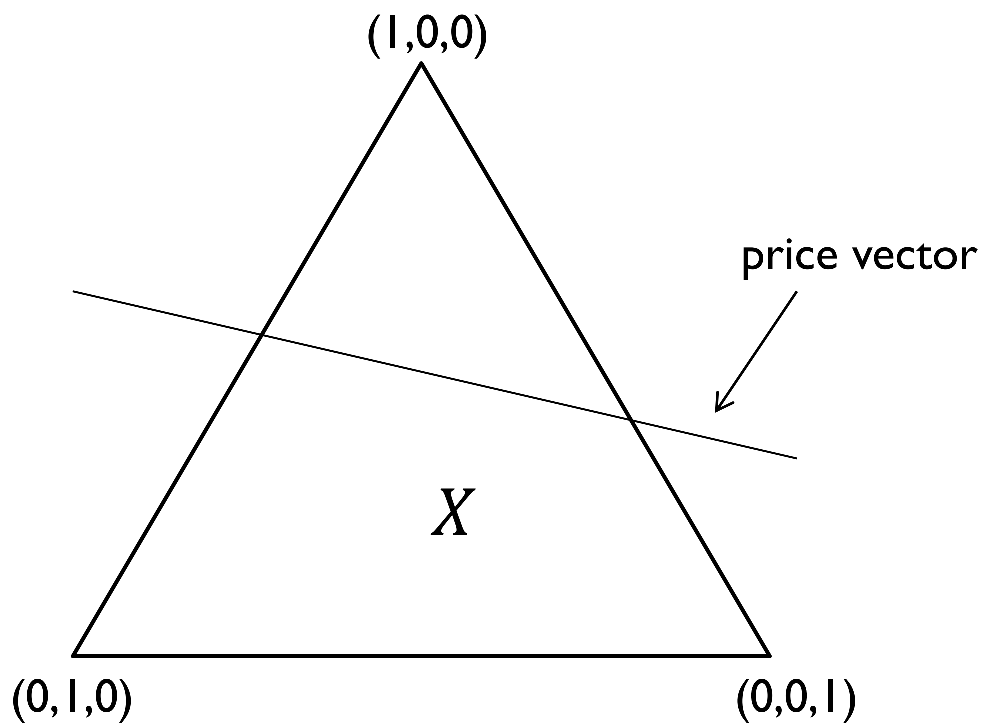
Proof idea

Step 1: An allocation rule is incentive compatible if and only if there exists a closed convex set X such that $x(u) \in \text{argmax}_{y \in X} u \cdot y$



Proof idea

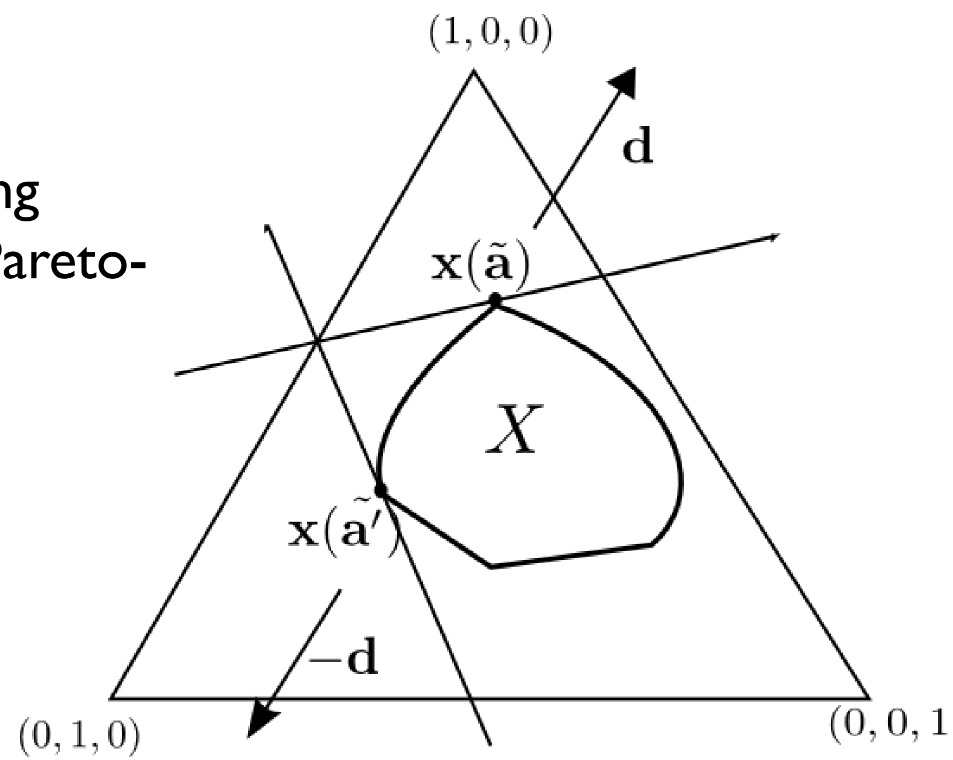
Step 2: Let X be the corresponding convex set for the allocation rule x. Enough to show that there exists a unique supporting hyperplane to X that intersects the interior of Δ.



Proof idea

Step 2: Let X be the corresponding convex set for the allocation rule x. Enough to show that there exists a unique supporting hyperplane to X that intersects the interior of Δ.

More than one supporting hyperplane contradicts Pareto-efficiency



Ordinal interim allocation rule

- Ordinal interim allocation rule for type t: maps ranking report of an agent of type t to assignment probabilities

$$x_t : \mathbf{\Pi} \rightarrow \Delta$$

Permutations of $\boldsymbol{S}$.

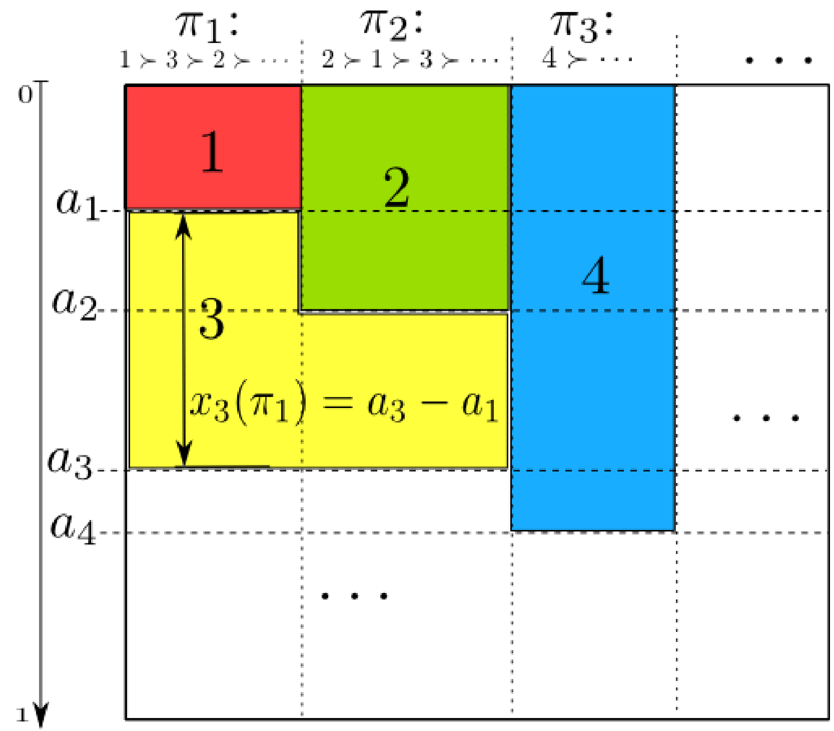
Feasible assignment probabilities

- Incentive Compatibility (IC):
  - $x_t(\pi)$ maximizes expected utility $v(\pi') = \boldsymbol{u} \cdot x_t(\pi')$
- Ordinal efficient within type:
  - Does not exist $x_t{}'$ with same average allocation as x_t but strictly Pareto improves in terms of first order stochastic dominance.
- "Valid": both IC and Ordinal efficient within type.

Characterization of valid allocation rules

Definition: Ordinal interim allocation rule $x: \Pi \to \Delta$ is **lottery-plus-cutoff** if there exists cutoffs $a_s \in [0,1]$ such that $x_{\pi(k)}(\pi) = \max_{j=1}^{k} a_{\pi(j)} - \max_{j=1}^{k-1} a_{\pi(j)}$.

Interpretation: agents have lottery numbers distributed Uniform(0,1). Can choose services s for which they do not exceed the cutoff a_s.



Theorem:

Every valid ordinal interim allocation rule is lottery-plus-cutoff.

Proof sketch

Lemma: An allocation rule $x(\pi)$is incentive compatible if and only if there exists a monotone submodular function $f: 2^{\{|S|\}} \rightarrow [0,1]$ s.t.

$$x_{\pi(k)}(\pi) = f(\{\pi(1), \dots, \pi(k)\}) - f(\{\pi(1), \dots, \pi(k-1)\})$$

Lemma idea

If x is incentive compatible, then for any $M \subseteq S$ one can define

$$f(M) = \sum_{j=1}^{|M|} x_{\pi(j)}(\pi) \quad , \quad \text{where } \{\pi(1), \ldots \pi(|M|)\} = M$$

f is monotone and submodular

Lemma idea (cont.):

If x is defined by $x_{\pi(k)}(\pi) = f(\{\pi(1), \dots, \pi(k)\}) - f(\{\pi(1), \dots, \pi(k-1)\}$ with monotone submodular f, the range of x is the vertex set of the base polytope of the polymatroid:

$$\sum_{s \in M} x_s \leq f(|M|) \quad \forall M \subseteq S$$

$$\sum_{s \in M} x_s = 1$$

$$x \geq 0$$

$\Rightarrow$ Greedy optimization with objective $u \cdot x$ (assuming $u_1 \geq u_2 \geq \cdots \geq u_{|S|}$) leads to set x's as we defined.

$\Rightarrow$ thus x is incentive compatible

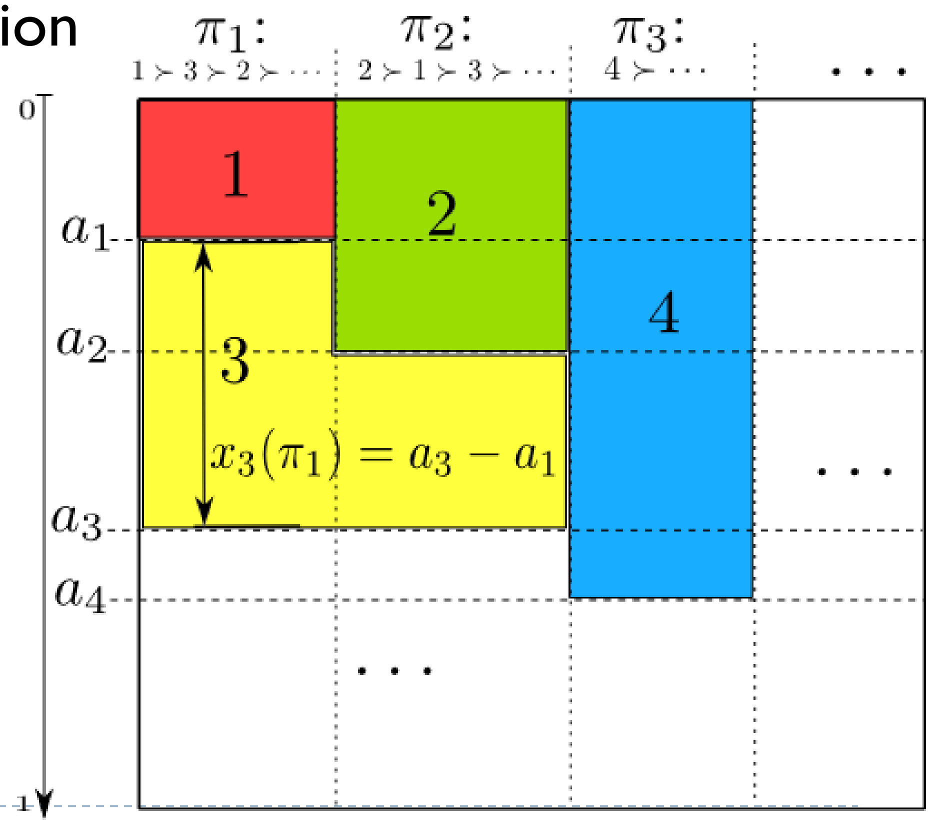
Exchange lemma:

Let f be a monotone submodular function corresponding to an incentive compatible allocation rule x.
If x is Pareto efficient, then for every $M_1, M_2 \subseteq S$

$$f(M_1 \cup M_2) = \max\{f(M_1), f(M_2)\}$$

Insights from ordinal mechanisms

- A valid mechanism is equivalent to assign a **menu** with all services with larger cutoffs than the lottery number: *randomized menu with nested menus*
- Only variables in the optimization problem are cutoffs



Solving the optimization problem

- Suppose public objective is a **linear combination of utilitarian and max-min welfare** (or any other linear objective)

$$W = \alpha \overbrace{\sum_t w_t v_t}^{\text{utilitarian welfare}} + (1-\alpha) \overbrace{\min_t v_t}^{\text{max-min welfare}}$$

parameter

arbitrary weights

expected utility of type t

- **Linear costs:** vector of cost $\boldsymbol{c}_{ts}$ for allocating an agent of type t to service s; budget $\boldsymbol{B}$ on expected costs. $\sum_t |n_t| x_{ts} \boldsymbol{c}_{ts} \le \boldsymbol{B}$

Optimization with randomized menus

$$\text{Max } W = \alpha \sum_{t,M} w_t v_t(M) z_t(M) + (1-\alpha) \min_t \sum_M v_t(M) z_t(M)$$

s.t.

Assign a menu $\sum_M z_t(M) = 1 \ \forall t$

Capacity $\sum_M n_t p_t(s, M) z_t(M) \le m_s \quad \forall s$

Budget $\sum_{s,t,M} n_t p_t(s, M) c_{ts} z_t(M) \le C$

$$\boldsymbol{z_t(M) \geq 0, \ \forall t, M \subseteq S}$$

Probability agent of type t chooses service s from M

Probability assigning menu M to type t

Too many variables!

Optimization with randomized menus

Utility prior F_t is "**logit**": if $u_{is} = a_{ts} + b_t \epsilon_{is}$ $\quad \epsilon_{is}$ i.i.d. standard Gumbel

Theorem: Under logit utility priors the optimal solution can be found in polynomial time.

Applying machinery to school choice

- Solve the optimal mechanism for the large market
- Translate the cutoffs from the opt mechanism to a finite market mechanism:

  construct priorities for each school and run

  Deferred Acceptance
- Under same budget for total miles bused, computed optimal for:
  - $\alpha = 1$: utilitarian welfare
  - $\alpha = 0$: max-min welfare
  - $\alpha = 0.5$: Equal weighting of above

Performance of opt under various α's

	$\alpha = 1$	$\alpha = 0.5$	$\alpha = 0$
Average expected utility	7.78	7.66	7.39
Expected utility for worse off type	2.52	7.39	7.39

Averages over 10,000 independent simulations

Elementary school choice in Boston (2012)

- Students rank any number of programs within their zone + walk-zone.
- Schools priorities students:
  1. Continuing
  2. Siblings
  3. Walk-zone (applies to only 50% of seats)
  4. lottery number
- Gale-Shapley's Deferred Acceptance Algorithm:
  1. Student applies to top choice.
  2. Program accepts if space available; otherwise bump out least priority student.
  3. Remove choice from bumped student; iterate.

Main entry grade K2:
- 77 schools
- 123 programs
- ≈20-60 seats/program

Data

- For each student, (of approximately 4000 students), have
  - Home location (14 neighborhoods, 868 geocodes)
  - Ranked list of preferences:
    - 1st choice, 2nd choice, 3rd choice, …
- For each school program, have
  - Location, test scores, demographics, program type, …

Modeling demand

- Multinomial logit:

$$u_{ts} = Q_s - D_{ts} + \omega \cdot \boldsymbol{W}_{ts} + \beta \epsilon_{ts}$$

utility quality distance Walk indicator "idiosyncratic taste" ~Gumbel(0,β)

- Fit Q, ω, β from micro choice data using MLE.

Q_s: 0-6.29 (additional utility in travelling distance)

ω: 0.86 (additional utility for walk zone)

β: 1.88 (standard deviation of taste shock)

Proposed Solution

- "**Home Based A (Baseline)**": Each family gets union of walk-zone, closest 2 top 25% schools, closest 4 top 50% schools, closest 6 top 75% schools, closest 3 "capacity schools."

- Logic:
  - Offer "enough" schools at various thresholds.
  - Compensate families in "bad areas" with more choices.
  - Dynamically adapts to changes in quality.
    - Unlike rigid zone maps.

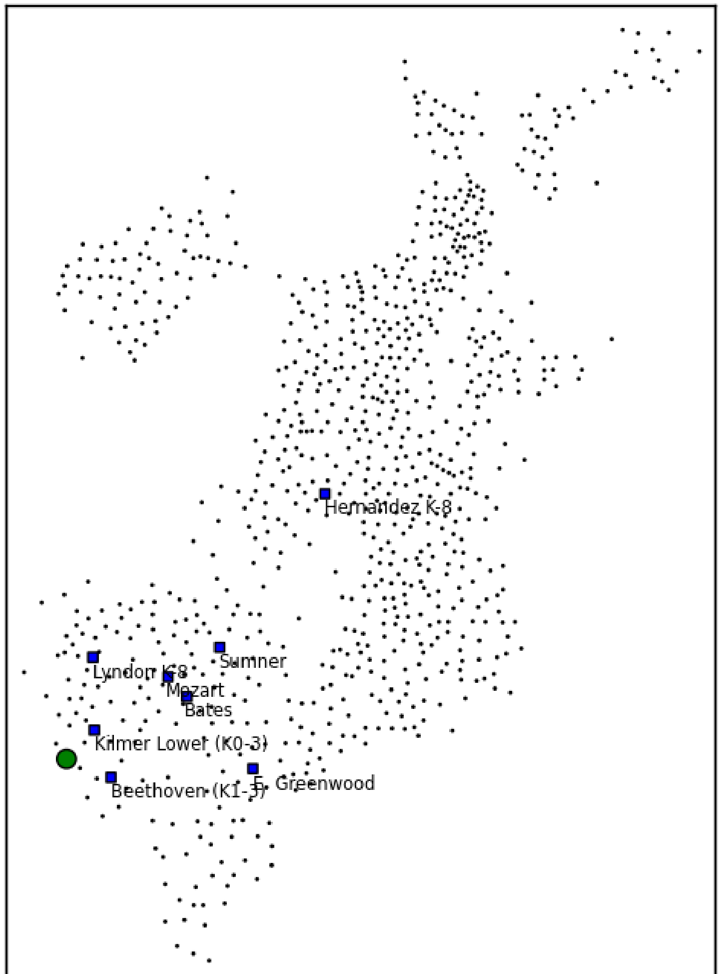
Evaluation of plans

	Minimum	Baseline	Opt	Opt A
Miles busing per student	0.35	0.64	0.71	0.63
Average exp utility	6.31	6.95	7.62	7.49
Exp utility for worse off type	2.86	4.53	7.05	7.02
% getting top choice	66	64	80	79
% getting 3rd choice	88	85	94	93

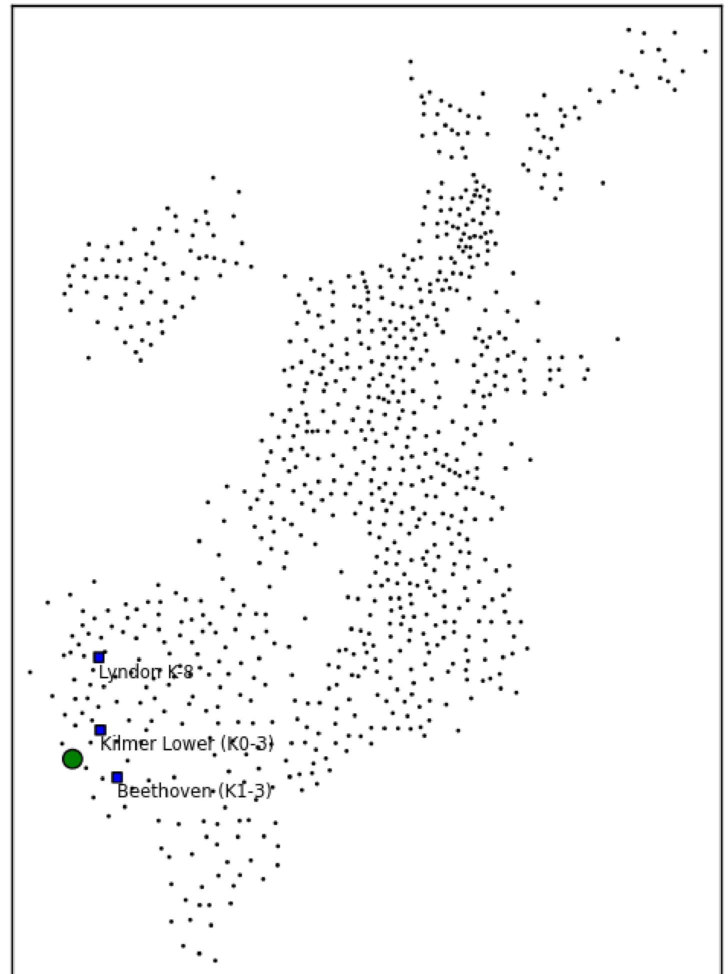
Averages over 10,000 independent simulations

Comparing Choice Menus for a Neighborhood Near "Higher Quality" Schools

Baseline

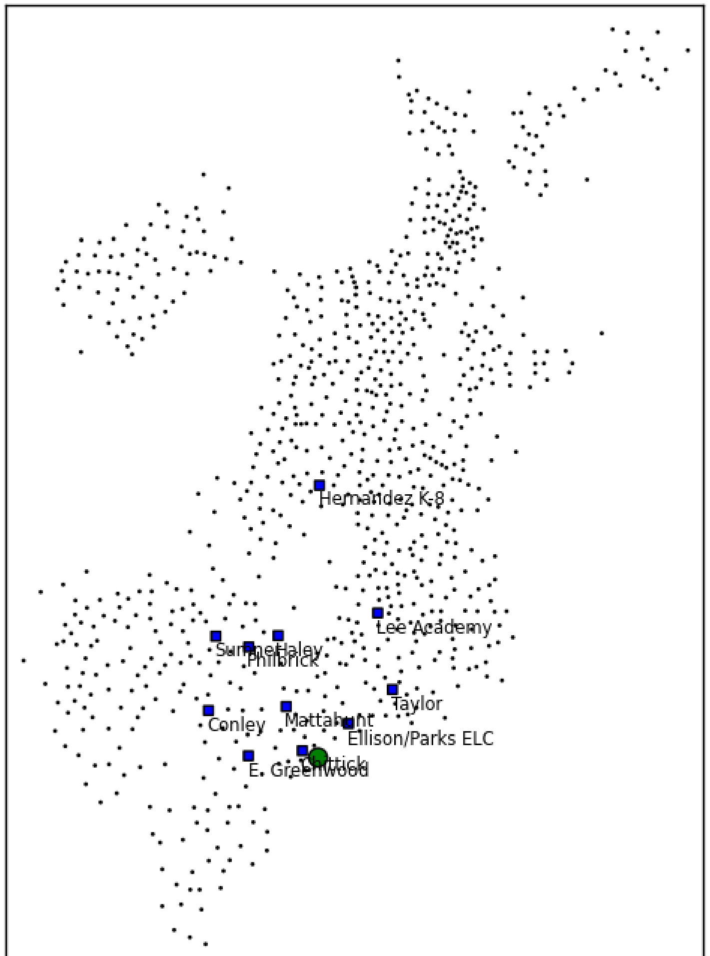


Opt A

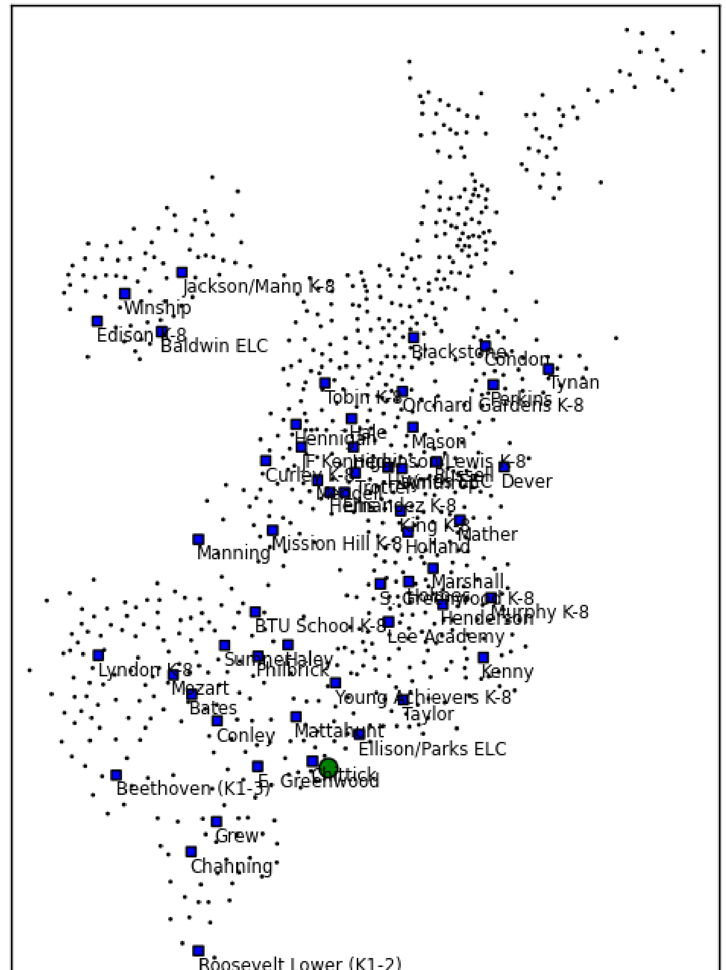


Comparing choice menus for a Neighborhood near “Lower Quality” Schools

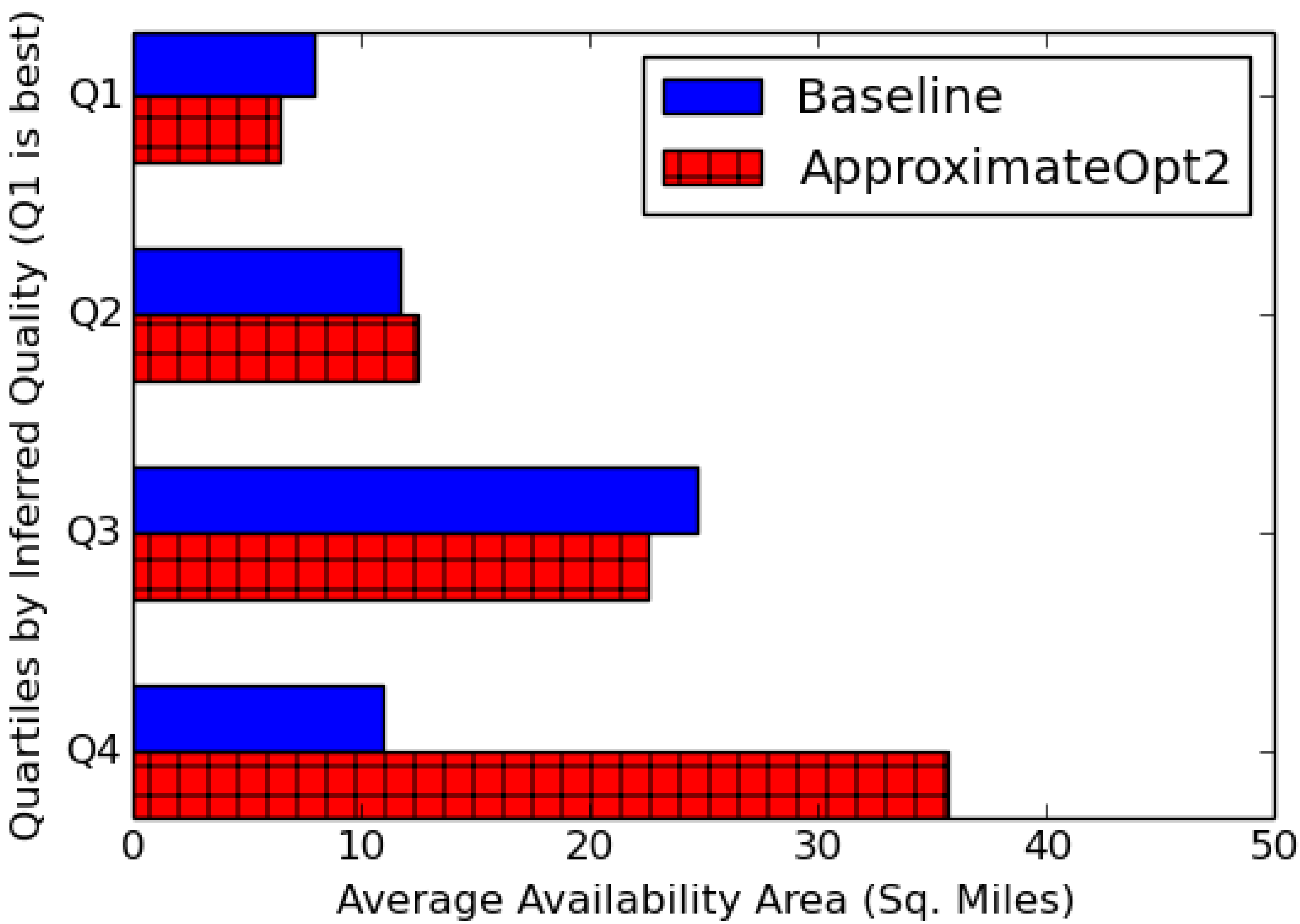
Baseline



Opt A



Optimal plan has larger catchment area for less popular schools



Summary of Insights

- **Possible to** simultaneously **achieve** high **efficiency**, **equity**, **predictability**, while staying within busing budget.
- **"Optimal plan" more aggressive** than Baseline compensating "lower quality" of choice with higher quantity.
  - Logic: families would only choose far away "lower quality" schools if they have a good reason, so offering to bus them to these schools is win/win.

Question: Can we improve community cohesion without affecting choice?

- "without affecting choice": same application process, same choices, **same assignment probabilities**.
- "community cohesion": conditional on being assigned, how many others from my community can I expect to be co-assigned with? Average this across students.
  - Proportional to # of same-community-pairs assigned together.

Ashlagi and Shi 2013: Improving community cohesion in school choice,

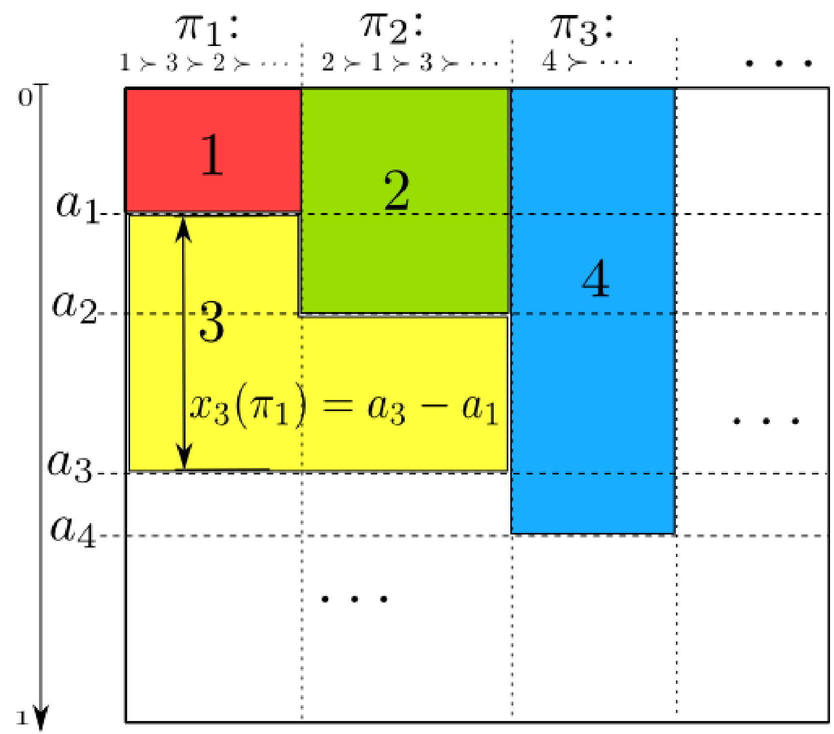
Characterization of valid allocation rules

Every valid ordinal interim allocation rule is lottery-plus-cutoff.

Flexibility:

a. lotteries (can correlate)

b. priorities



Maximize community cohesion

- z_{is}: random indicator for i being assigned to s.
- $c(i)$: community of student i.

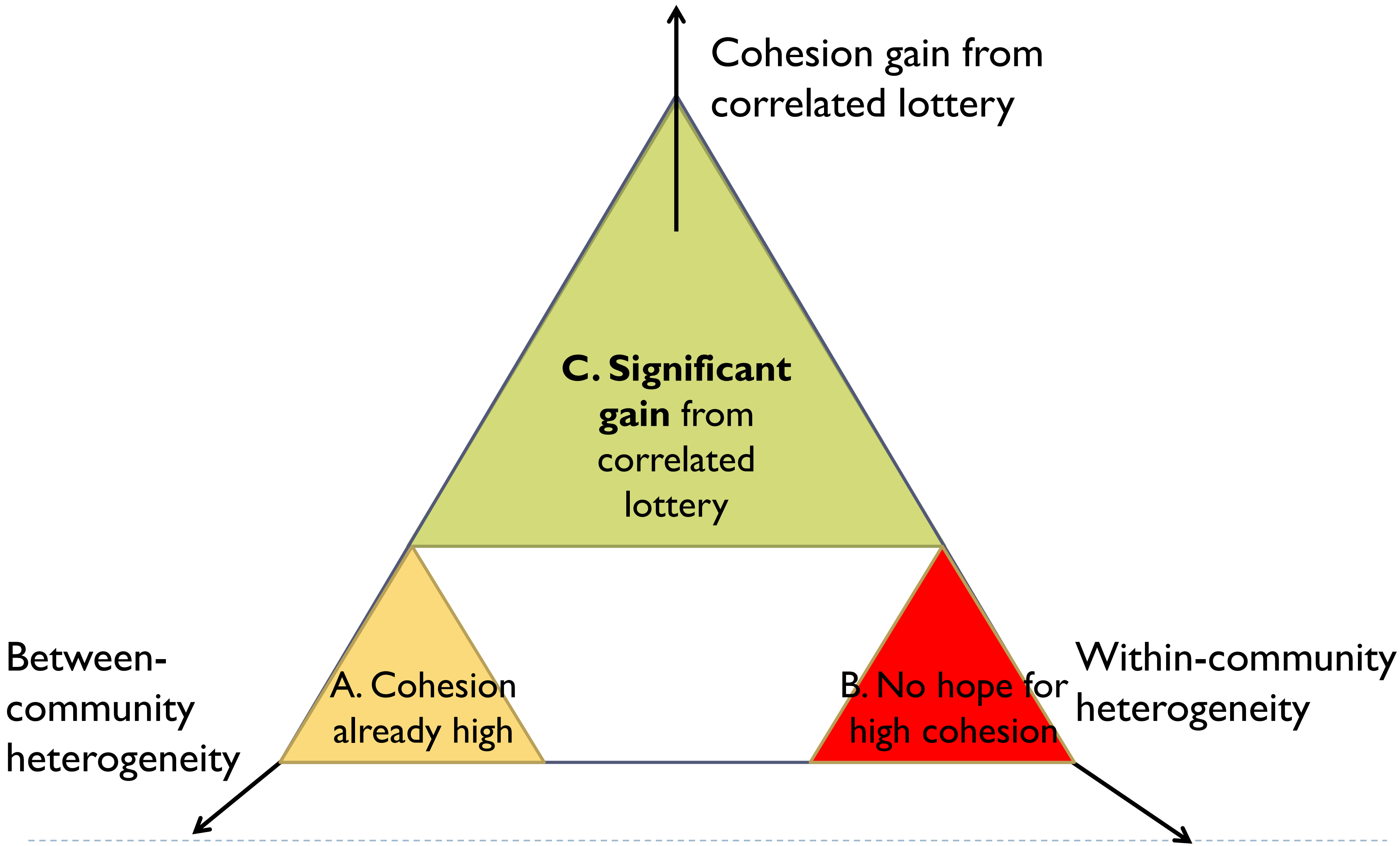
Max $\quad \sum_s E\left[\sum_{c(i)=c(i')} z_{is} z_{i's}\right]$ (community cohesion)

s.t. $\quad$ maintaining $p_{is} = E[z_{is}]$ for all i, s

z is feasible random assignment.

- Heuristic does well in simulation.

When can cohesion be improved?



No Lottery to Correlate for Continuing Students and Siblings

	K1		K2	
	% of students	**% assigned top choice**	% students	**% assigned top choice**
Continuing students	6%	**92%**	**47%**	**95%**
Non-continuing siblings	26%	**80%**	12%	**79%**
New families	68%	24%	41%	29%

So can only hope to significantly improve cohesion for new families.

Impact of Lottery Correlation

Grade	Student Type	Baseline	Correlated	Upperbound	Gain in Cohesion
K1	All	1.35	2.11	2.70	0.75
	Continuing	1.30	1.32	1.38	0.02
	Non-continuing siblings	1.35	1.43	1.56	0.08
	New families	1.36	2.44	3.26	**1.08**
K2	All	2.48	2.89	3.39	0.42
	Continuing	2.26	2.27	2.30	0.01
	Non-continuing siblings	2.91	3.01	3.23	0.10
	New families	2.61	3.58	4.69	**0.97**

For new families, **79% cohesion gain** over baseline for **K1** and **34%** for **K2**. Increase # of neighbors by ≈ 1.

Conclusion

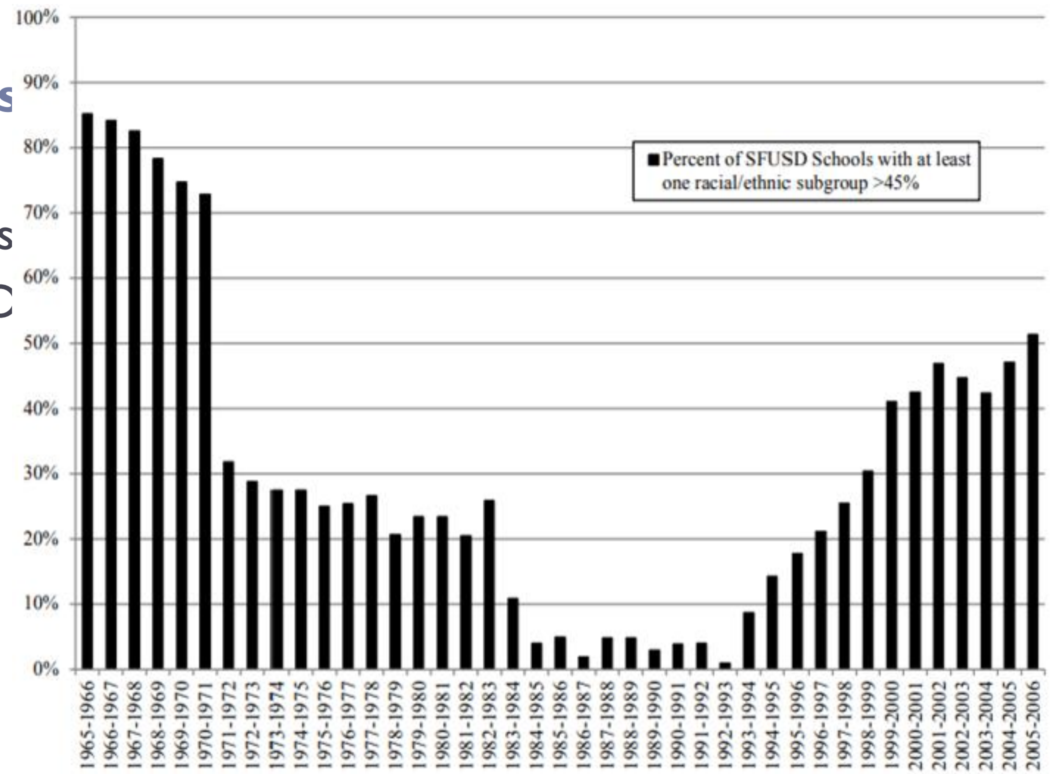
- Incorporate prior information into assignment problems
- Characterization of ordinal and cardinal incentive compatible Pareto optimal (within type) mechanisms in large markets
- Efficient computation of the ordinal mechanism in an relevant empirical environment
- Engineering approach for implementing in finite markets
- Open question: solve the cardinal mechanism for "realistic" preferences

SFUSD 2018 Board Resolution

- **2018 Board Resolution** (link)**:**
  - THEREFORE BE IT RESOLVED: The SFUSD will initiate a process to develop a new student assignment system, focusing on elementary schools, which will be predicated on **greater predictability**, **transparency**, **accessibility to neighborhood options**, **equity**, a strong commitment to **integrated schools**; and
  - FURTHER BE IT RESOLVED: In developing the policy goals for a revised student assignment system, staff will consider: Access to a **high quality school**; and Access to a **diverse school**; and Access to a **school where sibling(s) attend**; and
  - BE IT FURTHER RESOLVED: In developing a revised student assignment policy, staff will develop recommendations that will strive to: **Serve the needs of historically underserved students**; and Facilitate access to an elementary school within a **reasonable geographic distance and accessible to transit**; and ● Offer a **predictable, transparent and accessible** student assignment system.
- **Policy Goals:** Diversity (and integration), Predictability, Proximity, Equity of Access

Student Assignment in SF

- **Residential s**
  - Redlining
  - 1971: Horses
  - 1982-2002 C



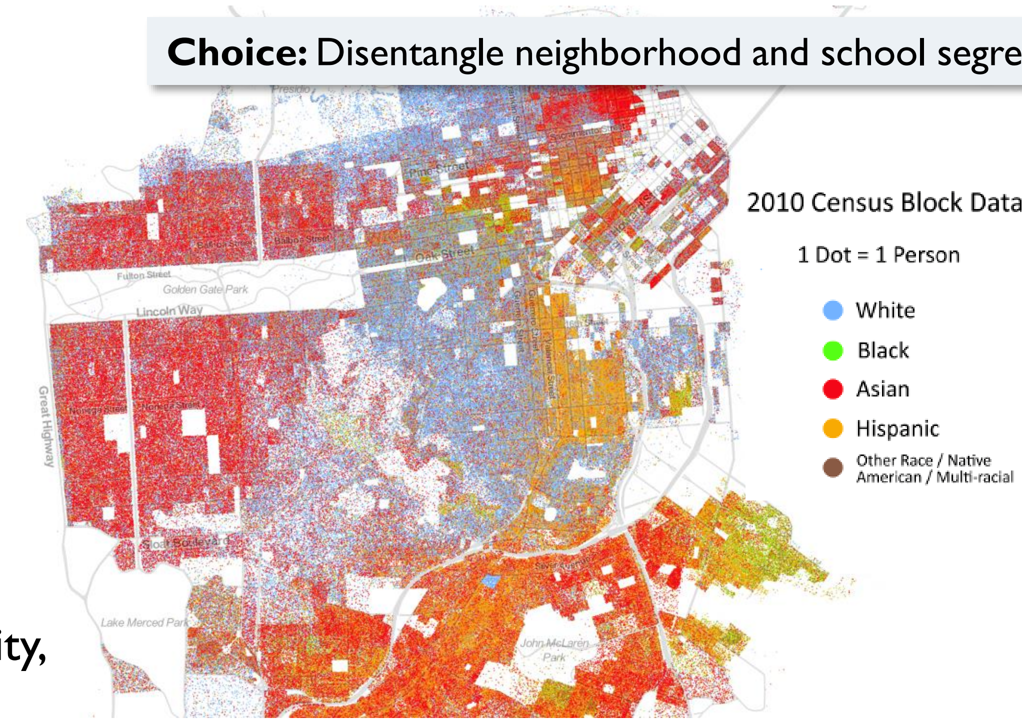
1971

Need for a New Student Assignment System in San Francisco

Problem:

Assigning students to public schools in San Francisco Unified School District (SFUSD)

Choice: Disentangle neighborhood and school segregation



Dec 2018: SFUSD Board of Education initiated a redesign of elementary student assignment

Goals: Diversity, Predictability, Proximity, Equity of Access

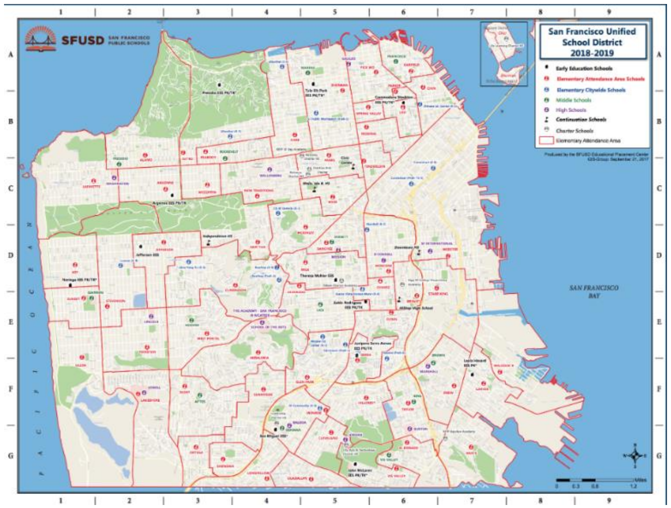
Image source: http://racialdotmap.demographics.coopercenter.org/

School Choice in Practice

- Deferred Acceptance (DA):
  NYC, Boston, Washington D.C., Denver, Seattle…

Since 2012 Home-Based Plan [Shi 12]

- Top Trading Cycles/ DA:
  San Francisco, New Orleans



School Choice in San Francisco: 2002-current

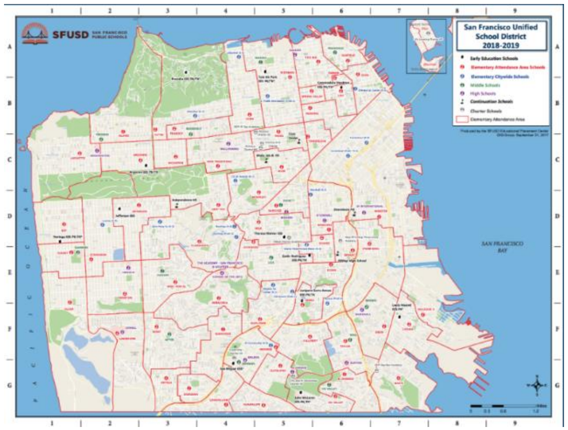
Elementary schools
~5,000 students, ~70 programs, ~50 schools

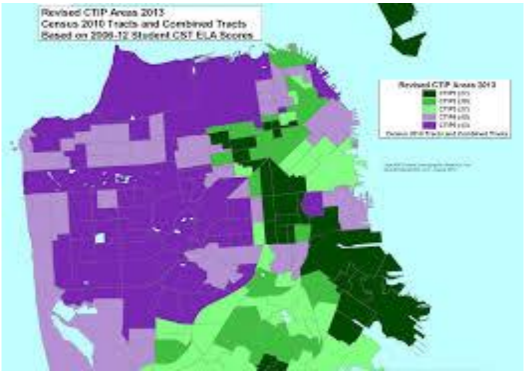
Families rank any number of programs

Students priorities at the schools:

1. Siblings
2. CTIP
3. Neighborhood
4. lottery number

Algorithm (2002-2018):
DA (Gale Shapley) followed by "trading cycles"
2019-present:
DA (Gale Shapley)





SFUSD Student Assignment: Goals and Challenges

- **Dec 2018:** SFUSD Board of Education initiated a redesign of elementary school student assignment
- **Goals:** Predictability, Proximity, Diversity, Equity of Access
- **Challenges**
  - SF residential segregation patterns (ethnic and SES)
  - Many programs and types of programs
  - Opt out to private/charter schools

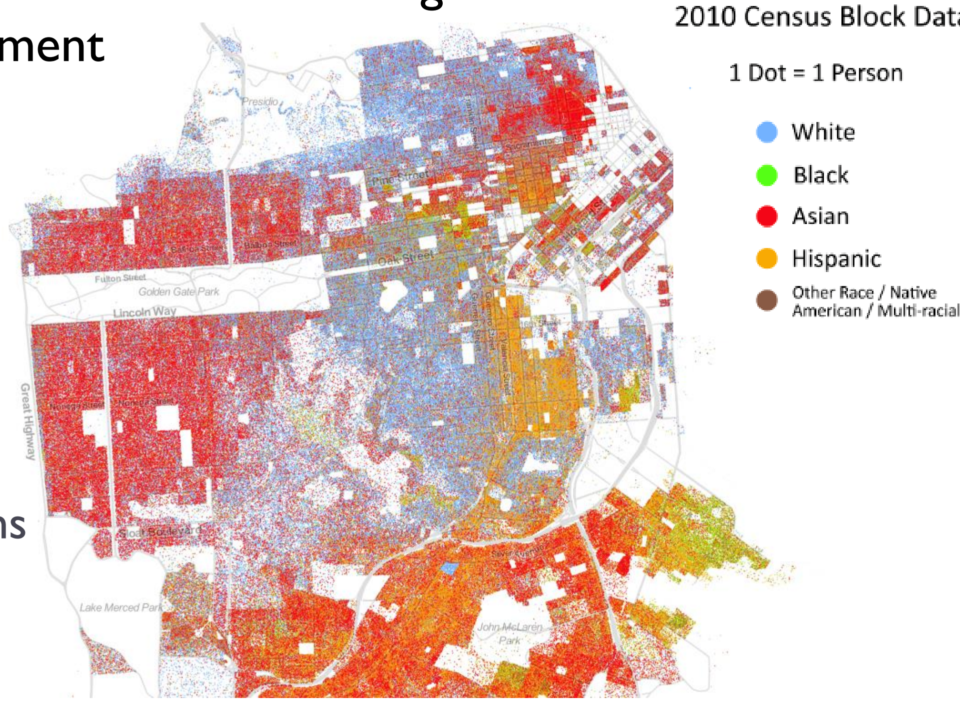


Image source: http://racialdotmap.demographics.coopercenter.org/

SFUSD Student Assignment: Policies in Practice

Idea 1: Neighborhood Assignment

- Students attend neighborhood school
- **Problem:** Racial + socioeconomic segr

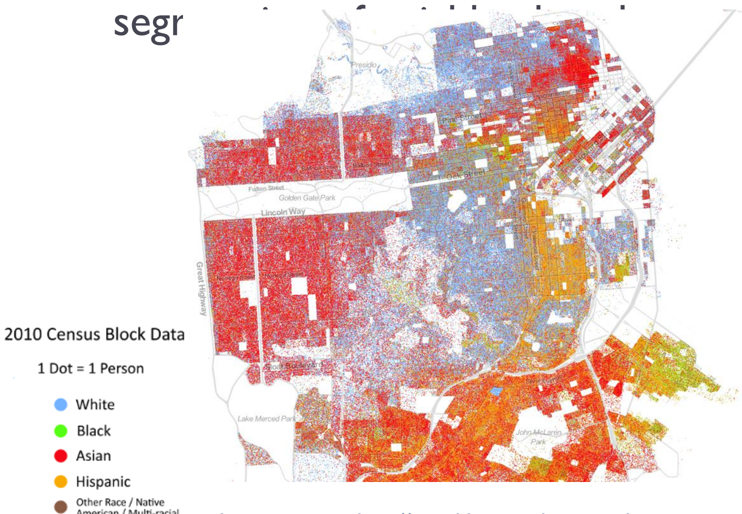
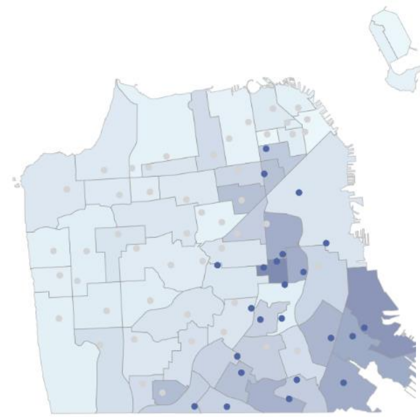


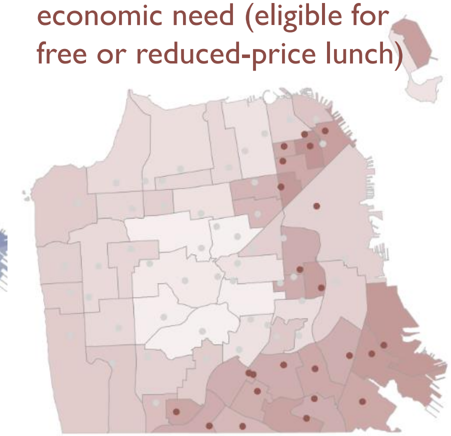
Image source: http://racialdotmap.demographics.coopercenter.org/

Idea 2: District-Wide Choice

- Students choose any schools, run DA or TTC
- **Problems:** Unpredictable and opaque, strategic issues, did not help with diversity

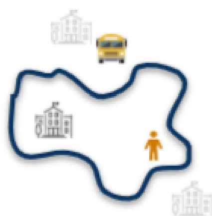
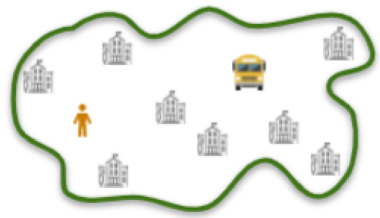


Schools with highest % historically underserved minorities (AALPI)



Schools with highest % students with socio-economic need (eligible for free or reduced-price lunch)

District Policy Concepts

	Concept #1: Initial Assignment + Choice	Concept #2: Choice in Small Zones	Concept #3: Choice in Medium Zones
Concept			
Geographic Constraints	Attendance Areas (1 school)	Zones (3 - 5 schools)	Zones (8-12 schools)
Student Assignment	Automatic assignment, then optional choice	Choice	Choice
Goals	Predictability, Proximity, Diversity	Predictability, Diversity, Proximity	Diversity, Predictability, Proximity

District Policy Concepts: Community Feedback

Community engagement meetings in Fall 2020

- Having **some** choice was important to most families, particularly AALPI and low-income families
- Families will find it easier to give feedback after having **specific zone boundaries**

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Concept			

General skepticism from AALPI and low-income families

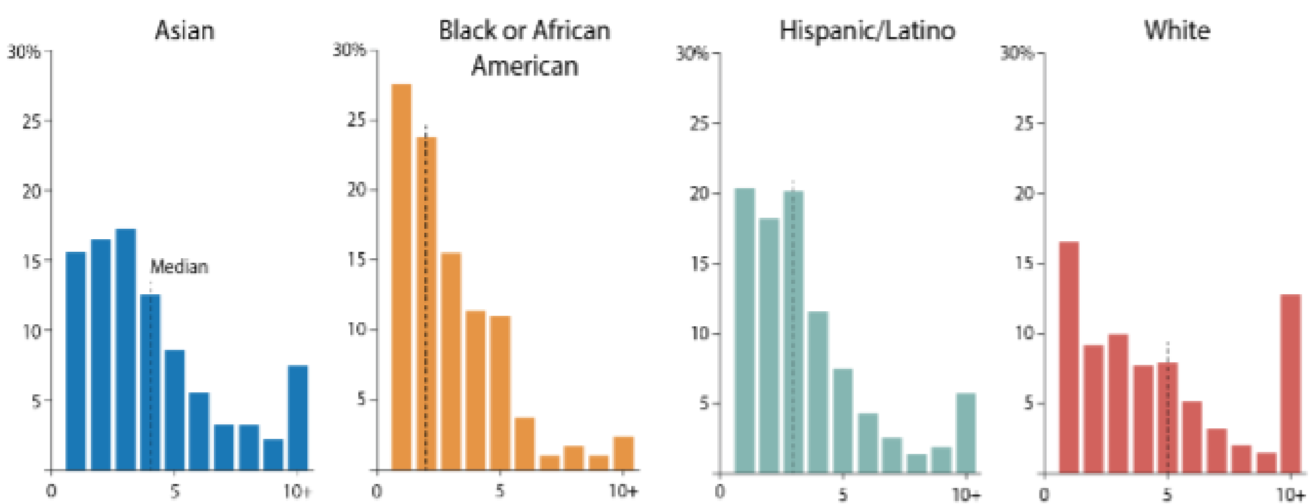
Popular only with high-income families & families in west SF

Popular amongst almost every demographic group

Unpopular due to concerns about feasibility and replicating district problems in each zone

List Lengths

Fraction of students with list of given length



List length

Discussion